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Identification of partially known models of the Susqueda hidroelectric power plant

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Abstract — This paper presents the identification of a hydroelectric power turbine dynamics. Knowledge of power plant behaviour is fundamental to obtain reliable and efficient operation of power systems. Starting from models already proposed, some modifications are suggested in order to adjust real plant response, recorded from different conditions and situations, to model behaviour.

Keywords — Identification. Hydroelectric Power Plant. Models.

I. INTRODUCTION

The AGC (Automatic Generation Control) performance of power systems is strongly influenced by the dynamic characteristics of its power plants. It is therefore of certain importance to have accurate models of the plants that contribute significantly to the AGC. There are in the literature some well-established structures for those models, but it is necessary, in each particular case, to identify the model parameters. In this paper the identification of a hydroelectric power turbine dynamic model has been performed for the Susqueda power plant, which is a hydroelectric plant belonging to the Endesa Group in Spain. Models taken as an initial step are described, although some modifications have been carried out in order to adjust the response of the real plant to model behaviour. The identification approach makes use a group of models with complex hydraulic dynamics, since all of them are nonlinear, and the models consider surge tank effects.

Dynamic behaviour of the power plant has been recorded in different conditions and situations, and consists of the gate opening and the measured electric power, which may be considered as the measured mechanical power generated by the turbine.

The identification has used registers that correspond to normal work conditions and they have been chosen for having the most complete frequency spectrum, hence they guarantee appropriate identification results.

This paper is organised as follows: Section II describes the physical characteristics of the power station. Section III presents the dynamic equations considering a general nonlinear model with surge tank effects, and proposes some adjustments of the model. Section IV shows the adjustment of the equations. Section V presents the results of the simulation using alternative models. Section VI describes a comparative study of the behaviour of the models. Finally, Section VII summarises the conclusions of this paper.

II. CHARACTERISTICS OF THE HYDROELECTRIC POWER PLANT

The Susqueda power station is situated next to Susquedas reservoir in the province of Girona (Spain), which is supplied by the river Ter. The total installed power is 86 MW with three units ($2 \times 37 \text{ MW} + 1 \times 12 \text{ MW}$) and with an annual production of 180 GWh.

This work deals with one of the 37 MW units of the power station. [Figure 1](#) presents a diagram of the main elements of the Susqueda power plant No 2 and shows the heads and flows that intervene in a hydroelectric plant model that considers surge tank effects.

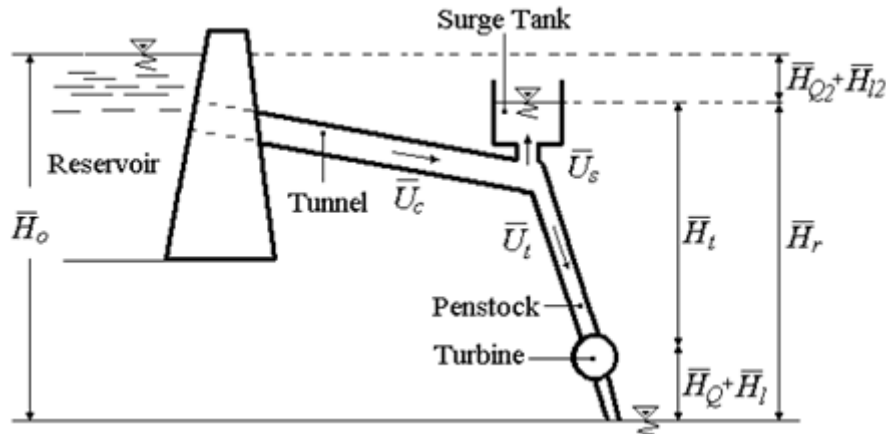



Fig. 1. Graphic of the hydroelectric power plant.

The hydroelectric power plant No 2 of Susqueda has the following characteristics ([Tables 1](#) and [2](#)):

Table 1: Plant Characteristics.

The Plant	
Total Head (H_{base})	174.41 m
Head Losses ($H_l + H_{l2}$)	10.39 m
Maximum Flow (Q_{max})	65 m ³ /seg
Installed Power	37 MW

Table 2: Characteristics of the conduits.

The Conduits	
Surge Tank (Cylindrical)	Height: $L_s = 100$ m Diameter: $\phi_s = 9$ m
Tunnel (Cylindrical)	Length: $L_c = 3500$ m Diameter: $\phi_c = 4.3$ m Internal Material: concrete.
	Length: $L_p = 250$ m Initial Diameter: $\phi_i = 4.3$ m Final Diameter: $\phi_f = 3.3$ m Internal Material: concrete.

III. GENERAL NONLINEAR EQUATIONS

The initial approach employed in this work is to consider the most general dynamic equations and tune the resulting model to match the reality. The general dynamic equations are taken from different models from (IEEE Working Group, 1992; Kundur, 1994; Quiroga and Riera, 1999; Quiroga, 2000). The general equations are:

Equation of continuity

$$\bar{U}_t = \bar{U}_c - \bar{U}_s. \quad (1)$$

Dynamics of the tunnel

$$\frac{d\bar{U}_c}{dt} = \frac{\bar{H}_0 - \bar{H}_r - \bar{H}_{l2}}{T_{WC}} = \frac{\bar{H}_{Q2}}{T_{WC}}, \quad (2)$$

$$\bar{H}_{l2} = f_{p2} \cdot \bar{U}_c \cdot |\bar{U}_c|. \quad (3)$$

Dynamics of the surge tank

$$\bar{H}_r = \frac{1}{C_s} \cdot \int \bar{U}_s \cdot dt - f_0 \cdot \bar{U}_s \cdot |\bar{U}_s|. \quad (4)$$

Dynamics of the penstock

$$\bar{H}_l = f_{p1} \cdot \bar{U}_t^2, \quad (5)$$

$$\overline{H}_t = \overline{H}_r - \overline{H}_l - z_p \cdot \tanh(T_{ep} \cdot s) \cdot \overline{U}_t = \overline{H}_r - \overline{H}_l - \overline{H}_Q \quad (6)$$

$$\overline{U}_t = \overline{G} \cdot \sqrt{\overline{H}_t} \quad (7)$$

Mechanical power

$$\overline{P}_{mechanical} = A_t \cdot \overline{H}_t \cdot (\overline{U}_t - \overline{U}_{NL}) \quad (8)$$

The variables and parameters and their meaning are described in [Table 3](#).

Table 3: List of variables and parameters.

Variables	Meaning
$\bar{H}_{(t,r,l,2,0)}$	Head in [pu] (t : turbine, r : surge tank, l : loss in the penstock, $l2$: loss in the tunnel, 0 : reservoir).
$\bar{U}_{(t,p,c,s)}$	Velocity of the water in the conduit or flow in [pu] (t : turbine, p : penstock, c : tunnel, s : surge tank).
$U U_{base}$	Velocity of the water in the conduit in [m/s] ($base$: normalised velocity).
Q_{base}	Base flow in [m ³ /seg]. $Q_{base}=Q_{max}$.
$H H_{(base)}$	Head in [m] ($base$: base value of head, i.e. the total available static head).
\bar{G}	Gate opening in [pu]
P_{elec}	Measured electrical power in [MW].
Parameters	Meaning
$A_{(p,c,s)}$	Cross section area of a conduit in [m ²] (p : penstock, c : tunnel, s : surge tank).
a	Wave velocity in [m/s].
g	Acceleration due to gravity [m ² /s].
α	$\alpha = \rho \cdot g(1 / \kappa + \phi / f \cdot E)$
ρ	Density of water [kg/m ³].
κ	Bulk modulus of compression of water [kg/(m · s ²)].
ϕ	Internal conduit diameter [m].
f	Thickness of pipe wall [m].
E	Young's modulus of elasticity of pipe material.
T	Surge tank natural period in [s]. $T=225$.
$T_{WP,WC}$	Water starting time at rated or base load in [s] (WP : penstock, WC : tunnel). $T_{WP}=0.82$, $T_{WC}=9.15$.
C_s	Storage constant of surge tank in [s]. $C_s=140$.
T_{ep}	Elastic time in [s] (ep : penstock). $T_{ep}=0.208$.
\bar{U}_{NL}	No load flow in [pu]. $\bar{U}_{NL} = 0.10$
$f_{p1,p2,0}$	Head loss coefficients in [pu] ($p1$: penstock, $p2$: tunnel, 0 : surge chamber orifice). $f_{p1}=0.0475$, $f_{p2}=0.089$, $f_0=0$.
A_t	Turbine gain in [pu]. $A_t=1.67$.
z_p	Hydraulic surge impedance of the conduit (p : penstock). $z_p=3.95$.

A. Formulas

The parameters for the power plant are deduced from values of [Tables 1](#) and [2](#) using the following formulas:

Elastic time in the penstock:

$$T_{ep} = L_p / a = L_p / \sqrt{g / \alpha}.$$

Water starting time in the penstock:

$$T_{WP} = \frac{L_p}{A_p \cdot g} \cdot \frac{Q_{base}}{H_{base}}.$$

Hydraulic surge impedance of the penstock:

$$z_p = T_{WP} / T_{ep}.$$

Water starting time in the tunnel:

$$T_{WC} = \frac{L_c}{A_c \cdot g} \cdot \frac{Q_{base}}{H_{base}}.$$

Storage constant of surge tank:

$$C_s = \frac{A_s \cdot H_{base}}{Q_{base}}.$$

Relationship between the normalised flow and the normalised water velocity in the tunnel or penstock:

$$\frac{Q}{Q_{base}} = \frac{A_{(p,c)} \cdot U}{A_{(p,c)} \cdot U_{base}} \Rightarrow \bar{Q} = \bar{U}.$$

[Table 3](#) also shows the values of the main parameters of the plant of Susqueda.

B. Block Diagrams

A block diagram for the general nonlinear model with surge tank effects is shown in [Fig. 2](#).

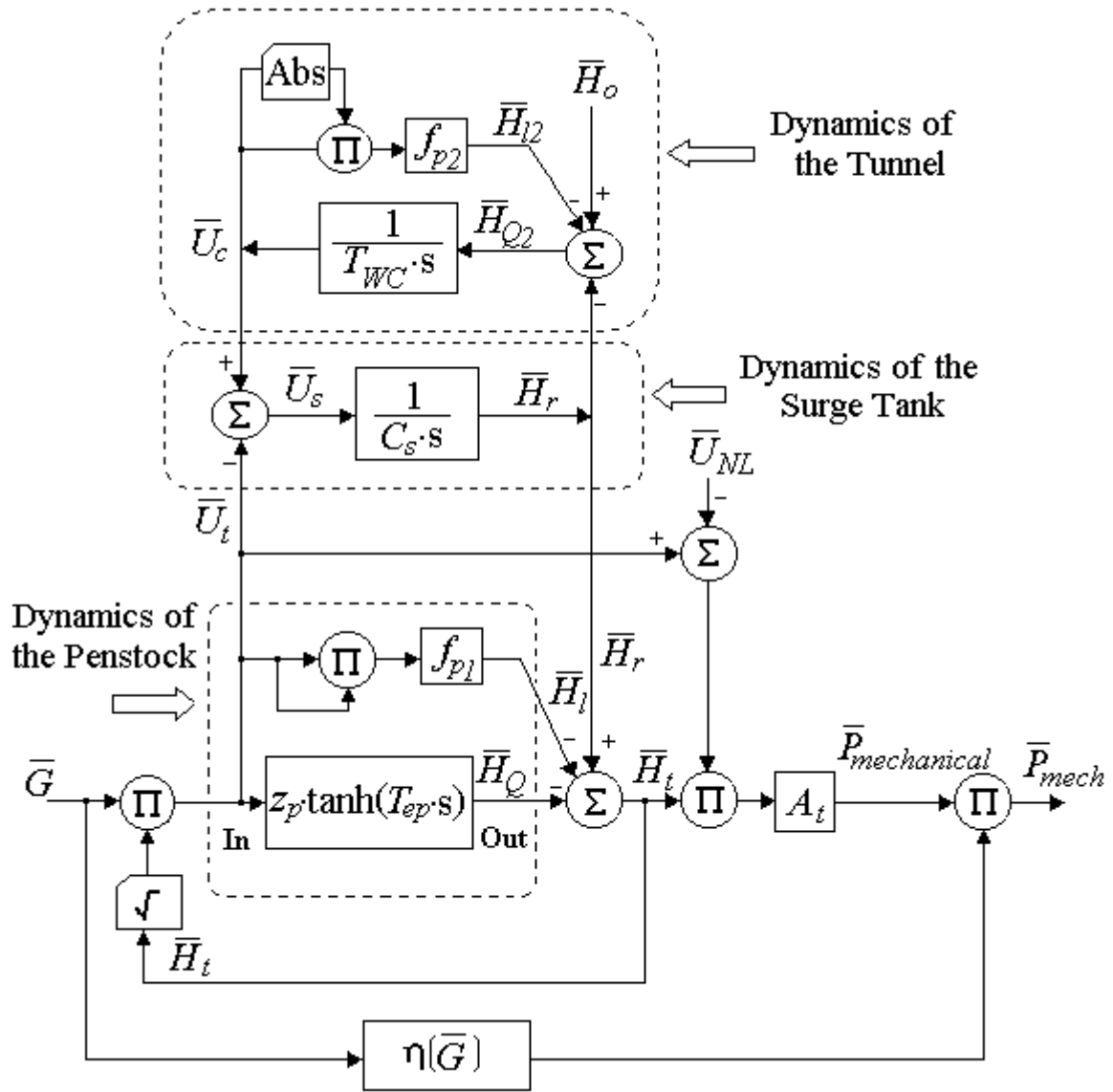


Fig. 2: Functional diagram for the general nonlinear model with surge tank effects.

To make the model equations useful, a key point is converting Eqn. (6) to a more treatable function. [Figure 3](#) depicts the blocks diagram employed to calculate the exact hyperbolic tangent function.

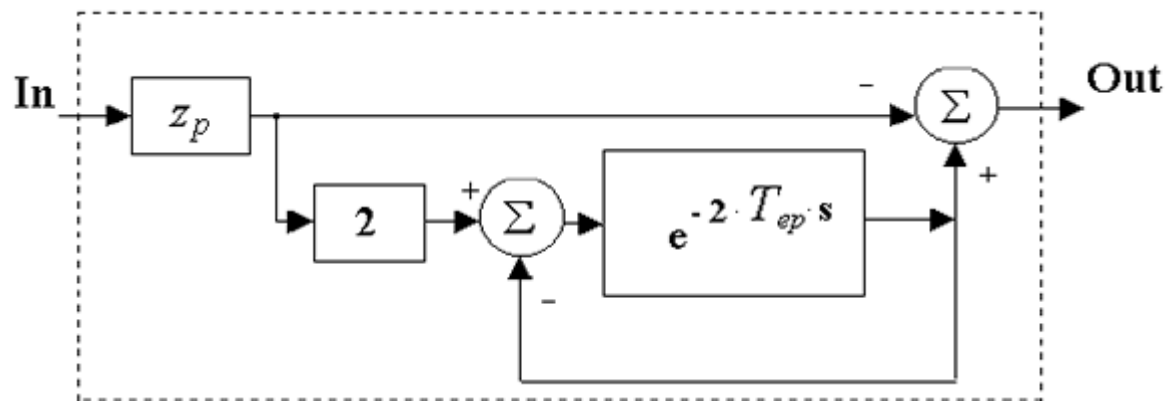


Fig. 3: Blocks diagram to calculate the expression $z_p \cdot \tanh(T_{ep} \cdot s)$.

The hyperbolic tangent can also be approximated by the alternative diagrams shown in Fig. 4.

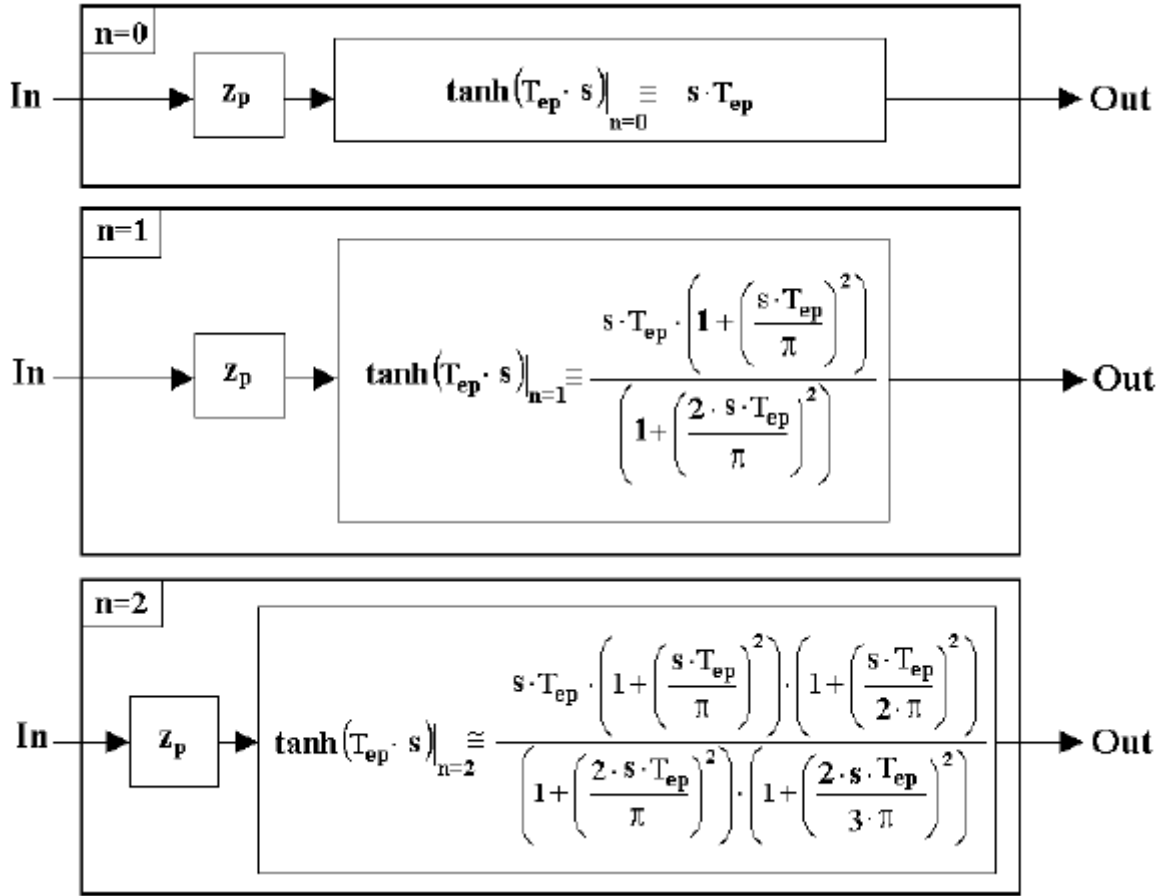


Fig. 4: Blocks diagrams used to calculate the approximations $n=0$, $n=1$ and $n=2$ of the hyperbolic tangent.

IV. ADJUSTMENT OF THE EQUATIONS

In order to match model behaviour to real plant response, two kind of adjustments must be performed. The first is needed to adjust the static gain of the model to the Susqueda plant. The second one is related with the pressure oscillations due to the surge tank effects.

A. Output power adjustment

To adjust the output power ($\overline{P}_{mechanical}$) to real values it is necessary to multiply the value given by Eqn. (8), by a nonlinear function $\eta(\overline{G})$ that represents the efficiency of the turbine. This function depends on the gate opening and its shape is similar to the efficiency curve of a Francis hydraulic turbine.

$$\overline{P}_{mech} = \eta(\overline{G}) \cdot \overline{P}_{mechanical}. \quad (9)$$

Table 4 gives the values of $\eta(\overline{G})$ deduced from steady state values of experimental registers considering that $P_{mech} = P_{elec}$. This last hypothesis holds because the efficiency of the generator is very high and can be considered almost constant for the whole range

of operation. The nonlinear function $\eta(\bar{G})$ is not considered in the general models given by (IEEE Working Group, 1992; Quiroga and Riera, 1999).

Table 4: Values of the function $\eta(\bar{G})$ for different gate positions deduced from experimental tests.

\bar{G} (pu)	P_{elec} (MW)	$\eta(\bar{G})$ (pu)
0.13	13.1	0.8922
0.18	13.6	0.9012
0.25	14.2	0.9107
0.36	14.5	0.9111
0.411	15.3	0.9113
0.603	27.3	0.9180
0.6635	30.15	0.9048
0.752	30.2	0.8410
0.8	30.75	0.8174
0.85	31.3	0.7874
0.896	31.8	0.7610

Moreover, the nonlinear function is calculated by means of the method of the least squares. The 1st, 2nd, 3rd and 5th order polynomials can be easily found using for example the function *polyfit* from the MATLAB mathematical package. This function finds the coefficients of a polynomial of degree m that fits the values given by the first and third column of [Table 4](#) in the least square sense. [Table 5](#) depicts these polynomials.

Table 5: Polynomials approximating $\eta(\bar{G})$.

Degree (m)	Polynomials	Norm (pu)
1 st	$\eta(\bar{G}) = -0.155147 \cdot \bar{G} + 0.95188$	0.1158
2 nd	$\eta(\bar{G}) = -0.654517 \cdot \bar{G}^2 + 0.5181708 \cdot \bar{G} + 0.8258124$	0.0402
3 rd	$\eta(\bar{G}) = -0.7932113 \cdot \bar{G}^3 + 0.569734 \cdot \bar{G}^2 - 0.026849177 \cdot \bar{G} + 0.8895742$	0.0287
5 th	$\eta(\bar{G}) = 158112 \cdot \bar{G}^5 - 3905104 \cdot \bar{G}^4 + 347891776 \cdot \bar{G}^3 - 141045755 \cdot \bar{G}^2 + 26587286 \cdot \bar{G} + 0.7179617$	0.0128

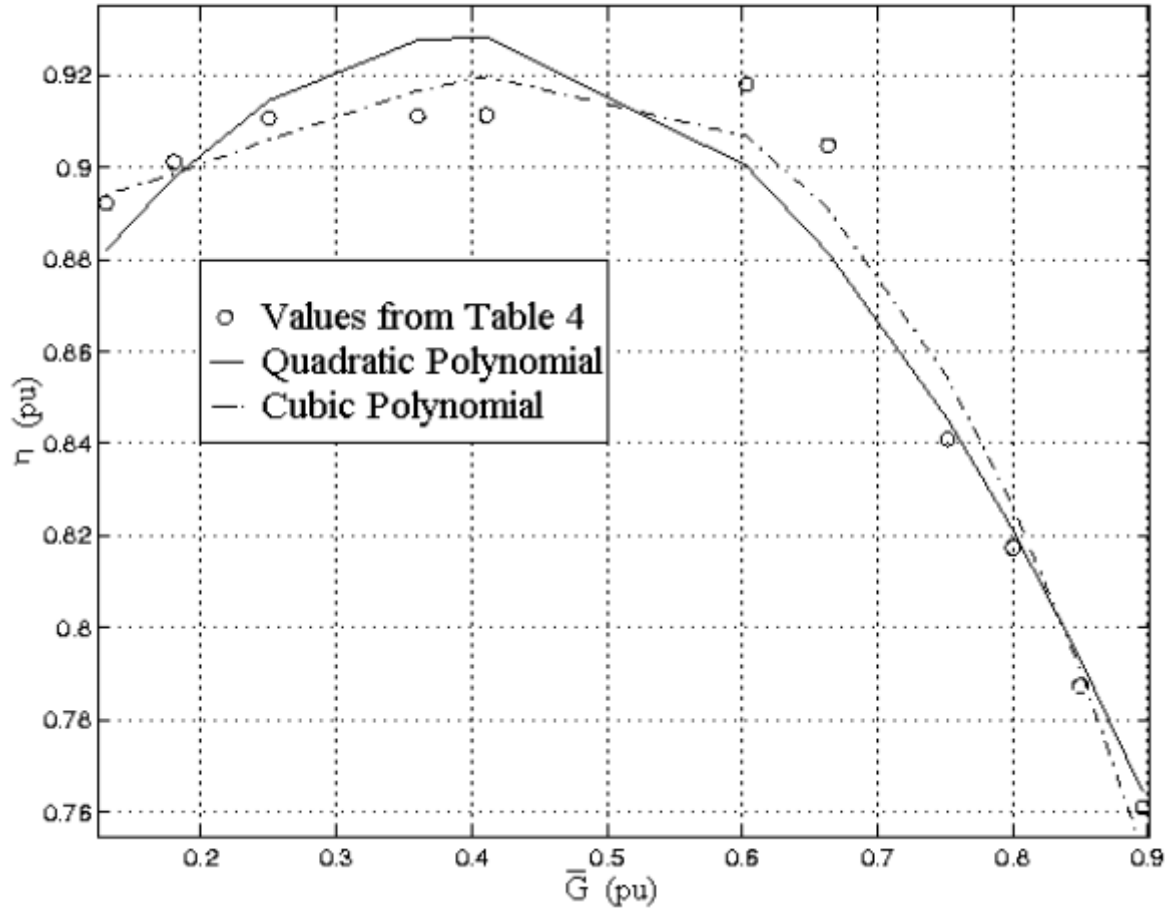


Fig. 5: Plot of the nonlinear function $\eta(\bar{G})$ calculated by the quadratic and the cubic polynomials.

[Figure 5](#) shows the quadratic and the cubic polynomials that represent $\eta(\bar{G})$ versus the steady state values taken from the third column of [Table 4](#).

B. Surge tank natural period

The period (T) of the pressure waves due to the surge tank is a function of three physical parameters of the hydroelectric plant: the cross section area of the surge tank (A_s), the cross section area of the tunnel (A_c) and the length of the tunnel (L_c).

$$T = 2 \cdot \pi \cdot \sqrt{\frac{L_c \cdot A_s}{g \cdot A_c}} \quad (10)$$

In order to adjust the oscillation period of the model with the period of the real plant it is necessary to slightly adjust the model parameters. Since the surge tank in Susqueda is cylindrical and very large, there exists the possibility that the diameter of the surge tank could vary a small percentage along its 100m of height.

Therefore, its diameter ϕ_s may be used to adjust the surge tank natural period (T). The modified value of the diameter is only five per cent less than the original diameter. The physical parameters ϕ_c or L_c (diameter and length of the tunnel) may also be used to adjust T ; however, the modified values for these parameters are greater than the five per

cent obtained for ϕ_s . Apart from this, the surge tank has not got an orifice; for this reason the value of the loss coefficient f_0 is equal to zero.

V. IDENTIFICATION RESULTS

[Table 6](#) presents a group of four nonlinear models for the power plant. These models differ in the way the block "In-Out" is implemented.

Table 6: Description of the Models A.

Derived Models	Reference Models (Nonlinear models with surge tank effects)
Models A Given by Eqns. (1) to (7) and Eqn. (9).	<p>A1). Elastic water column in the penstock and non-elastic water column in the tunnel. Hyperbolic tangent calculated by means of the "In-Out" block of Fig. 3.</p> <p>A2). Elastic water column in the penstock and non-elastic water column in the tunnel. Hyperbolic tangent calculated by means of the "In-Out" block of Fig. 4, approximation $n=2$</p> <p>A3). Elastic water column in the penstock and non-elastic water column in the tunnel. Hyperbolic tangent calculated by means of the "In-Out" block of Fig. 4, approximation $n=1$.</p> <p>A4). Non-elastic water columns. Hyperbolic tangent calculated by means of the "In-Out" block of Fig. 4, approx. $n=0$.</p>

[Figure 6](#) shows the measured electric power labelled as $P_{mech}(\text{register})$, and the mechanical power calculated by means of the Models A. [Figures 7](#) and [9](#) show the gate opening, the measured electric power $P_{mech}(\text{register})$ and the mechanical power P_{mech} (Models A).

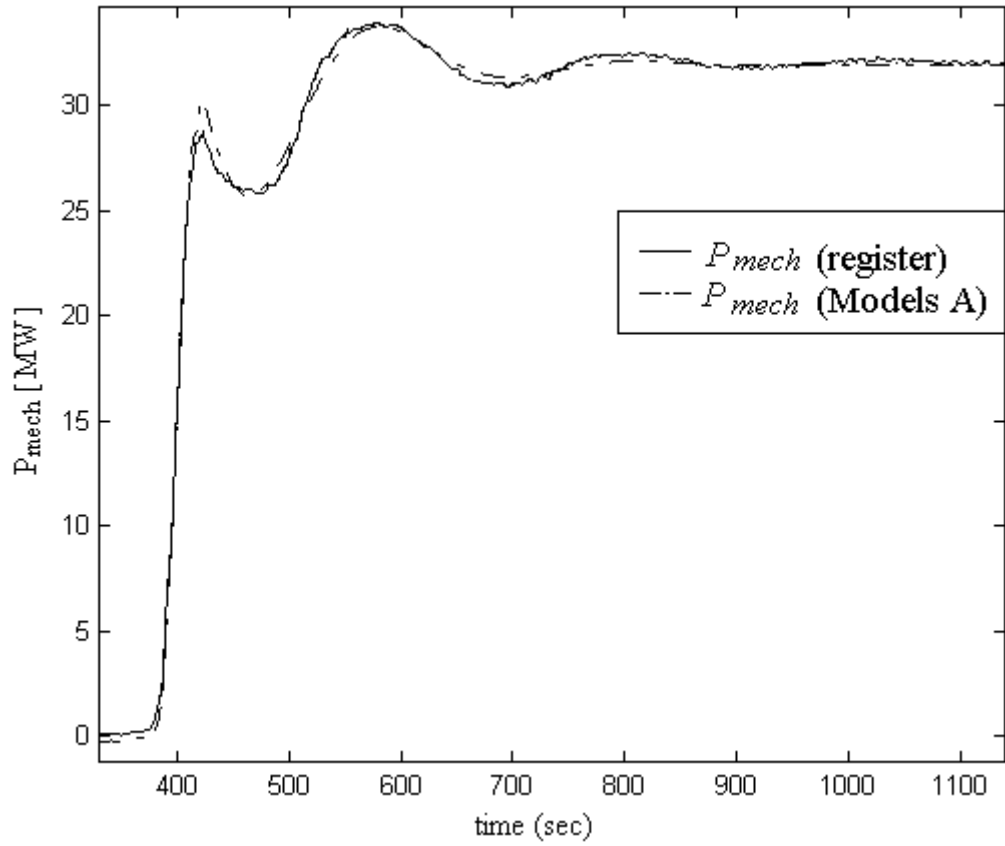


Fig. 6: Identification of Susqueda using the Models A.

In [Figs. 6, 7, 8, 9, 10](#) and [11](#) P_{mech} (Models A) represent the same curves for Models A1, A2, A3 and A4 since the differences are indistinguishable, however, more precise details are given in Section VI.

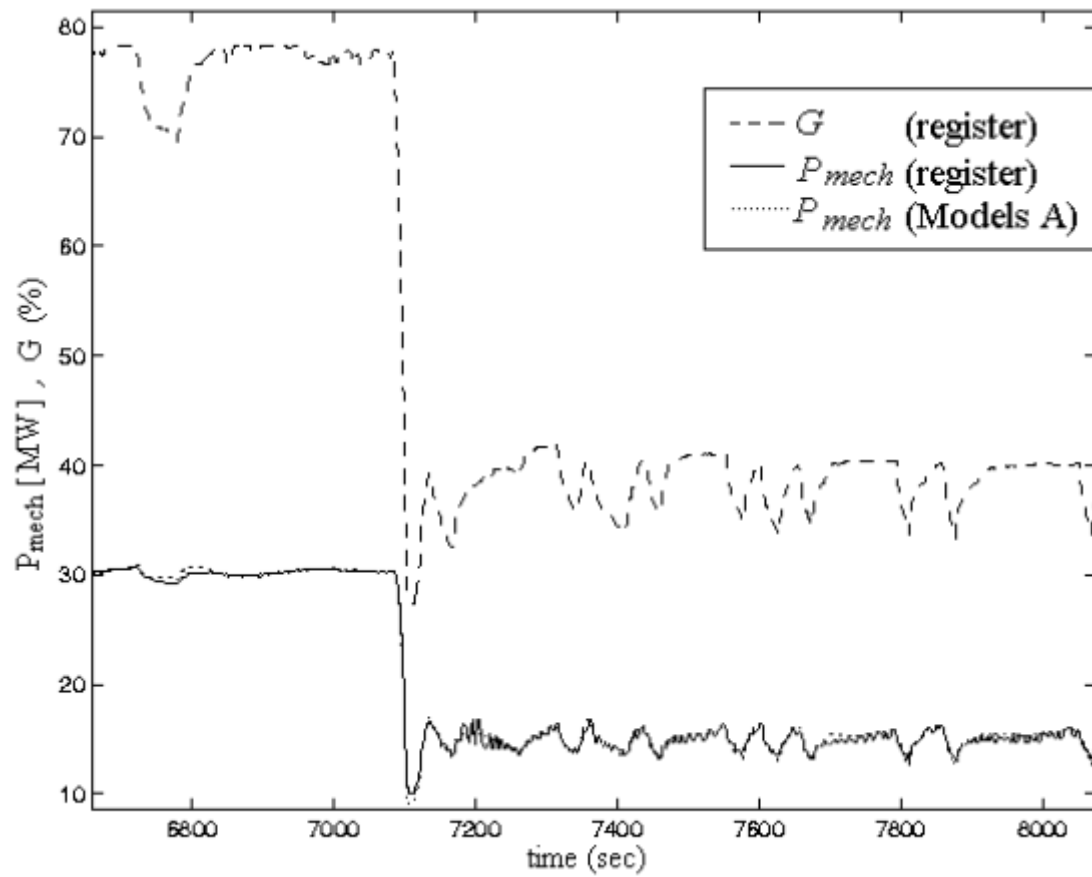


Fig. 7: Identification of Susqueda using the Models A.

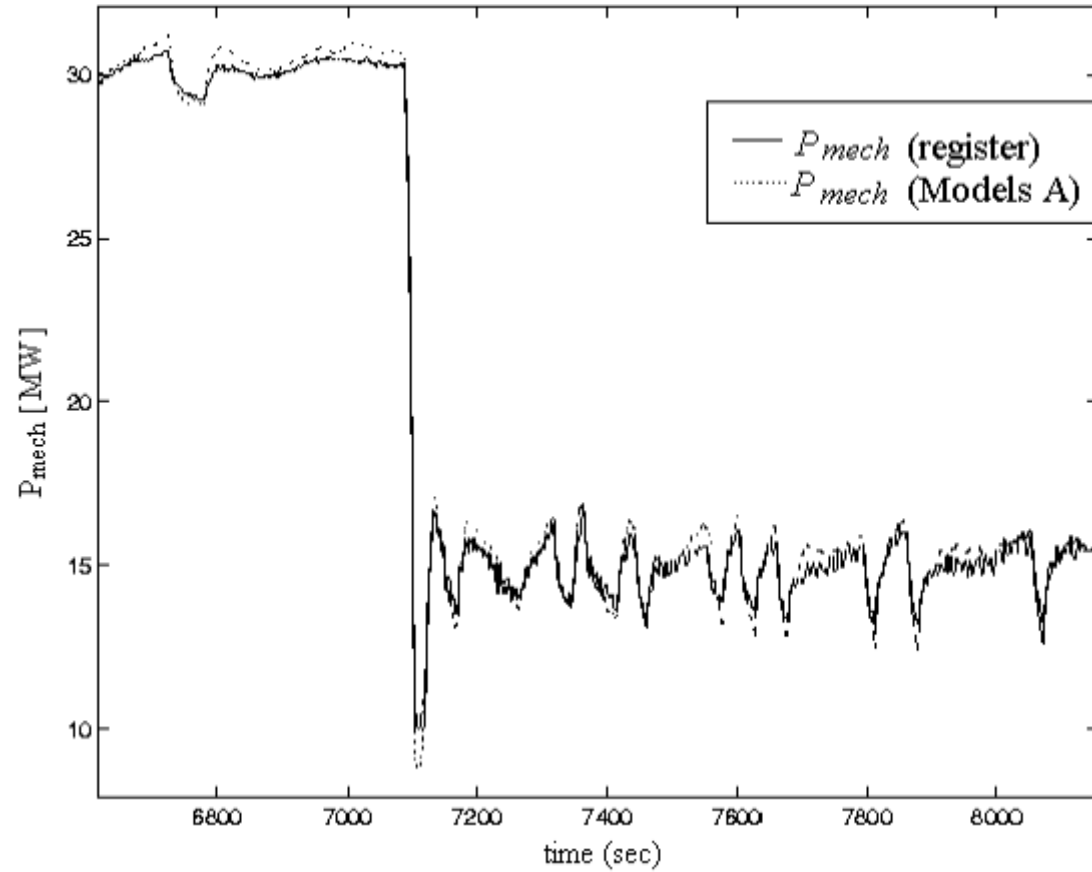


Fig. 8: Detail of [Fig. 7](#).

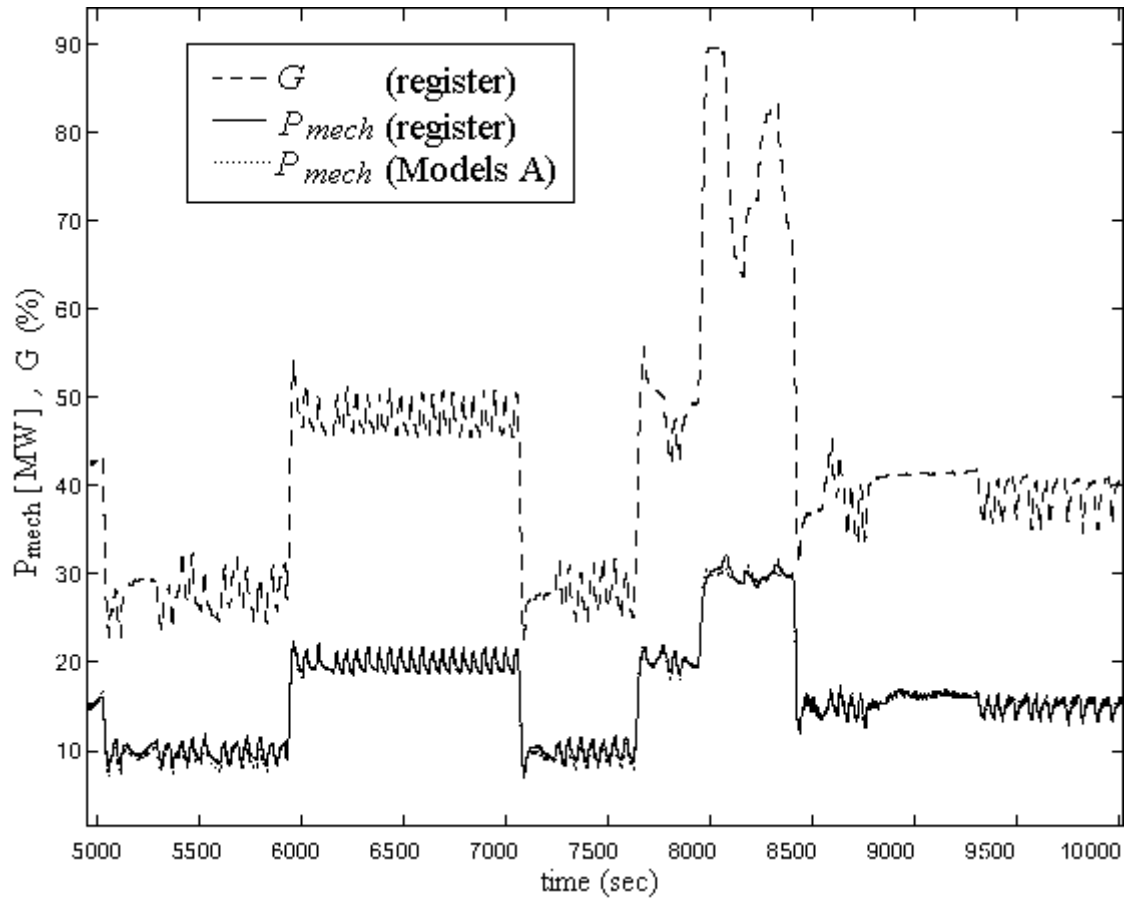


Fig. 9: Identification of Susqueda using the Models A.

[Figures 8](#) and [10](#) show the detail of [Figs. 7](#) and [9](#), respectively.

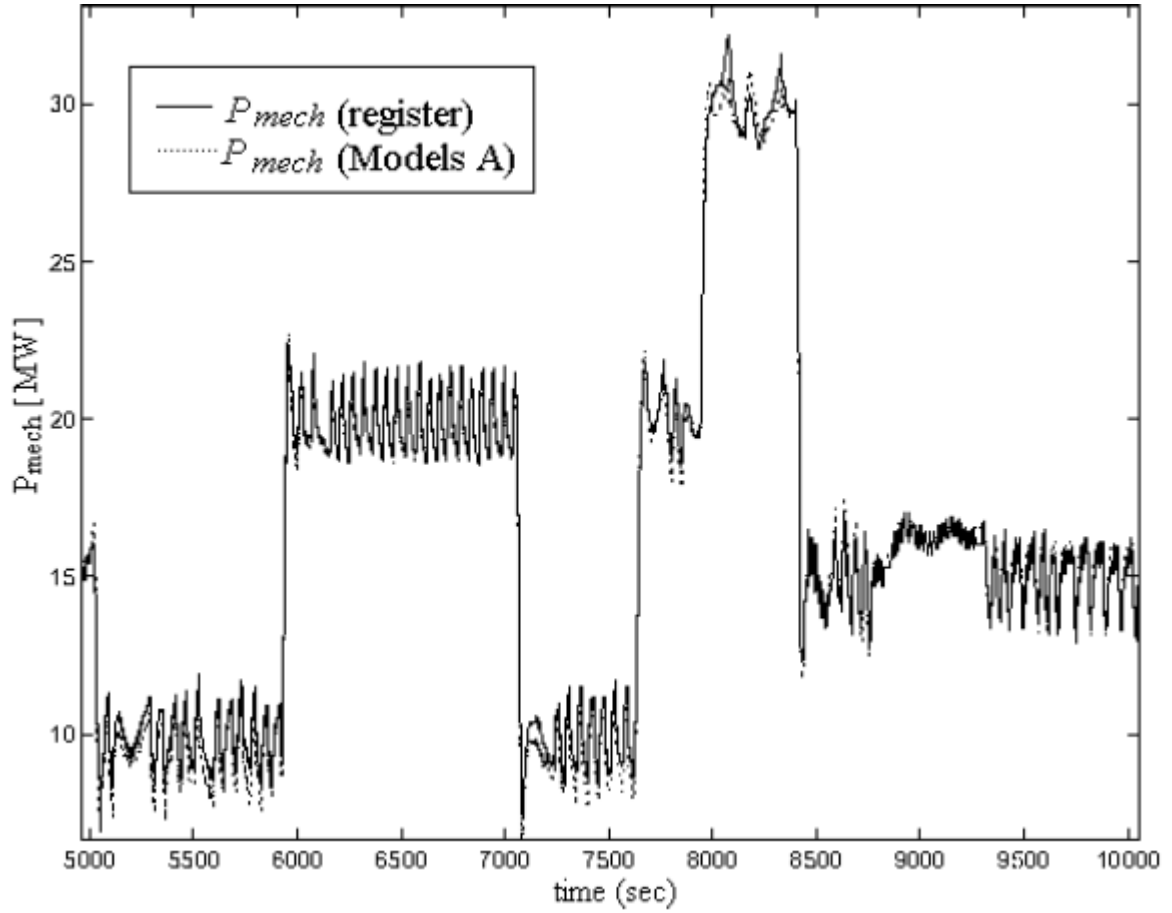


Fig. 10: Detail of [Fig. 9](#).

In [Figs. 6, 7, 8, 9, 10](#) and [11](#) the degree $m=5$ of the polynomial $\mathcal{V}(\bar{G})$ is used.

VI. COMPARATIVE STUDY

This section proposes a comparative study of the responses of the models, although the four models give good identification results of the Susqueda hydropower plant. A quadratic error index is defined by means of the following expression:

$$Quadratic \ Error = \sum_0^N \left(\bar{P}_{mech(Re\ gister)} - \bar{P}_{mech(ModelAx)} \right)^2,$$

where Ax represents the models A1, A2, A3 or A4.

The register chosen to calculate the quadratic error is depicted in [Fig. 11](#), where the hydraulic system reaches the steady state value after a variation in the gate opening. [Table 7](#) describes the quadratic error of the models for polynomials with degree 2 and 5, approximating the nonlinear function $\mathcal{V}(\bar{G})$.

The comparative study between the real records and the response of the models shows that the difference in the value of the quadratic error is between 0.15 to 0.39 for the Models A, giving quite a good representation of the real plant. Models A2, A3 and A4

have a quadratic error value that is a sixty per cent greater than the value of the quadratic error of the model A1, while the differences among the models A2, A3 and A4 are around 0.1 per cent.

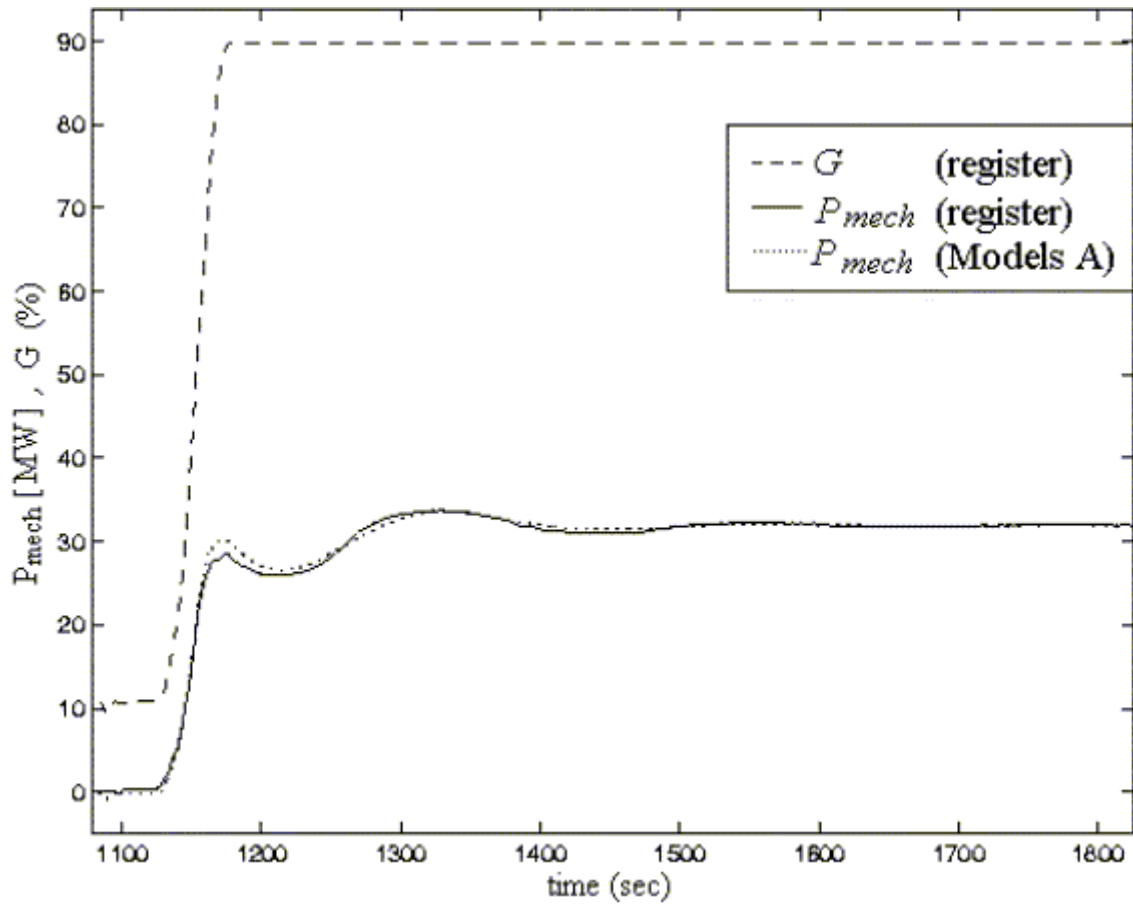


Fig. 11: Identification of Susqueda using the Models A.

Table 7: Quadratic error found using the models A, where the nonlinear function $\eta(\bar{G})$ is approximated by two polynomials with degree 2 and 5.

Models	Quadratic Error: for the polynomial of $\eta(\bar{G})$ (m = 5)	Quadratic Error: for the polynomial of $\eta(\bar{G})$ (m = 2)
A1	0.1530	0.1554
A2	0.3830	0.3878
A3	0.3832	0.3885
A4	0.3835	0.3891

Model A1 has the least quadratic error between 0.1530 (for m=5) and 0.1554 (for m=2) and thus the best approximation to the real power plant, although the models A2, A3 and A4 are also good approximations.

VII. CONCLUSIONS

This paper has presented an identification process of the hydroelectric power station of Susqueda using the models A. These models are based on models taken from (IEEE Working Group, 1992 and Quiroga and Riera, 1999; Quiroga, 2000), and have been adjusted in order to match the power plant behaviour.

The identification process follows two adjustment procedures. The first one tunes the static gain of the Susqueda power plant by taking into account a nonlinear function $\eta(\bar{G})$ in the calculation of the mechanical power. The shape of this function is similar to the efficiency curve of a Francis turbine.

The second one adjusts the oscillation time period (T), caused by the surge tank, by a five per cent modification of the measured value of the surge tank diameter.

The comparative study shows on one hand that the models A1, A2 and A3 need more complex expressions to calculate the hyperbolic tangent. On the other hand, the model A4 is the easiest to simulate since it considers non-elastic water columns and the hyperbolic tangent function is represented by means of a simple derivative function. Furthermore, the model A4 may be expressed as a nonlinear system in the state space and thus may be used in the nonlinear controller design.

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